Geometry I, 2010: Mid-term test

Last name:

First name(s):

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $\mathbf{a} = \begin{pmatrix} -3 \\ -2 \\ 6 \end{pmatrix}$. Determine $|\mathbf{a}|$.

2. Let $A = (-3, 3, -2)$ and $B = (-3, 4, 0)$, and let P be one third of the way along the line segment from A to B. (That is $|\overrightarrow{AP}| = \frac{1}{3}|\overrightarrow{AB}|$.) Determine the position vector $\mathbf{p}$ of P.

3. Determine parametric equations for the line through the points $(-3, 3, -2)$ and $(-2, 5, 0)$.

4. Determine the cosine of the angle between the vectors $\mathbf{p} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. Simplify your answer as much as possible.

5. Apply Gaußian elimination to transform the following system of linear equations into echelon form. [You do not need to determine the solutions of the system.]

$$\left.\begin{aligned} y + 5z &= -5 \\ x - 2y + 5z &= -5 \\ -2x + 4y + 5z &= -5 \end{aligned}\right\}.$$

6. Determine the intersection, as a set of points, of the plane defined by the equation $x - 2y + 2z = 3$ with the plane defined by $-y + 3z = 2$. [Your answer must be correctly specified as a set to obtain a mark.]

7. Calculate the area of the parallelogram determined by the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}$.

8. Determine a nonzero vector parallel to the plane defined by

$$3x + y - 3z = 17.$$

9. Exactly which of the following statements are true? [Your answer must be completely correct to obtain a mark.]

(a) If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are vectors such that $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{v}$ is orthogonal to $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{w}$.

(b) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ implies that $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

(c) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ for all vectors $\mathbf{v}$ implies that $\mathbf{u} = \mathbf{0}$.

(d) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

(e) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

10. Let A, B, C be points in 3-space with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively, and let D be the point of 3-space such that $ABCD$ is a parallelogram.

Determine an expression for the position vector $\mathbf{d}$ of D in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$.