Geometry I, 2009 : Mid-term test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $\underline{a} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. Determine $|\underline{a}|$.

2. Let $A = (2, 1, -2)$ and $B = (1, -3, 4)$, and let P be the mid-point of the line segment AB. Determine the position vector $\underline{p}$ of P.

3. Determine Cartesian equations for the line through the point $(2, 1, -2)$ in the direction of the vector $\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$.

4. Determine the cosine of the angle between the vectors $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$.

5. Apply Gaussian elimination to transform the following system of linear equations into echelon form. [You do not need to determine the solutions of the system.]

$$\begin{cases} x & + & 3y & + & 2z = & -2 \\ & & & & 2z = & -2 \\ 2x & + & 5y & + & 2z = & -2 \end{cases}.$$

6. Determine the intersection, as a set of points, of the plane defined by $x - 3y + z = 5$ with the plane defined by $y + 2z = 1$. [Your answer must be correctly specified as a set to obtain a mark.]

7. Calculate $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$.

8. Determine a non-zero vector parallel to the plane defined by

$$2x + y - 4z = -12.$$

9. Exactly which of the following statements are true? [Your answer must be completely correct to obtain a mark.]

(a) $\underline{u} \times (\underline{v} + \underline{u}) = \underline{u} \times \underline{v}$ for all vectors $\underline{u}, \underline{v}$.

(b) $\underline{u} \cdot (\underline{v} \times \underline{u}) = 0$ for all vectors $\underline{u}, \underline{v}$.

(c) $\underline{u} \times \underline{v} = \underline{v} \times \underline{u}$ for all vectors $\underline{u}, \underline{v}$.

(d) $(\underline{u} \times \underline{v}) \times \underline{w} = \underline{u} \times (\underline{v} \times \underline{w})$ for all vectors $\underline{u}, \underline{v}, \underline{w}$.

(e) $(\underline{u} \times \underline{v}) \times \underline{w} \neq \underline{u} \times (\underline{v} \times \underline{w})$ for all vectors $\underline{u}, \underline{v}, \underline{w}$.

10. Let O be a fixed origin in 3-space, let P and Q be any points in this space, with respective position vectors $\underline{p}$ and $\underline{q}$, and let R be the point such that $OPQR$ is a parallelogram.

Determine an expression in terms of $\underline{p}$ and $\underline{q}$ for the free vector represented by the bound vector $\overrightarrow{PR}$.