Geometry I, 2008 : Mid-term test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $\underline{u} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. Determine the vector of length 1 in the same direction as $\underline{u}$.

2. Let $A = (1, -2, 3)$, $B = (2, -1, -1)$. Determine the position vector of the point P on the line segment AB, such that $|\overrightarrow{AP}| = \frac{1}{3}|\overrightarrow{AB}|$.

3. Let $A = (3, 3, 1)$, $B = (3, -1, 4)$. Determine a vector equation for the line through A and B.

4. Calculate $\begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$.

5. Determine a vector orthogonal to the plane defined by $3x+y-4z = -12$.

6. Apply back substitution to determine all solutions of the following system (in echelon form) of linear equations in x, y, z:

$$\begin{cases} 2x & - & 3y & - & z & = & -1 \\ & & & & 2z & = & -4 \end{cases}.$$

7. Calculate $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$.

8. Determine a Cartesian equation for the plane Π through $(1, 1, 1)$, such that the intersection of Π and the plane defined by $-3x + 2y + 4z = 12$ is the empty set.

9. Suppose $\underline{u}$ and $\underline{v}$ are non-zero non-parallel vectors. For each of the following statements, say whether it is true or false. [Your answers must all be correct to get a mark.]

(a) $\underline{v} \cdot (\underline{u} \times (\underline{u} \times \underline{v})) > 0$

(b) $\underline{v} \cdot ((\underline{u} \times \underline{v}) \times -\underline{u}) > 0$

(c) $\underline{v} \cdot ((\underline{u} \times \underline{v}) \times \underline{u}) > 0$

(d) $\underline{u} \cdot (\underline{v} \times \underline{u}) > 0$

(e) $\underline{u} \cdot (-\underline{v} \times \underline{u}) > 0$

10. Suppose $ABCD$ is a parallelogram, such that $\overrightarrow{AC}$ represents $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

and $\overrightarrow{BD}$ represents $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. Determine the vector represented by $\overrightarrow{AB}$.

[Hint: First draw a parallelogram $ABCD$ to help you think.]