Geometry I, 2007 : Mid-term test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $A = (-3, 1, 1)$, $B = (2, -1, 3)$. Determine the vector represented by $\overrightarrow{AB}$.

2. Let $\underline{u} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$. Determine the vector of length 1 having direction opposite that of $\underline{u}$.

3. Determine Cartesian equations for the line through the point $(-3, 1, 1)$ and in the direction of the vector $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

4. Determine the cosine of the angle between the vectors $\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

5. Determine a Cartesian equation for the plane through the point $(-3, 1, 1)$ and orthogonal to the vector $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

6. Determine all solutions of the following system of linear equations in x, y, z:

$$\begin{cases} x & - & y & - & z & = & -2 \\ -3x & + & 3y & + & 4z & = & 7 \\ 2x & - & 3y & + & z & = & -2 \end{cases}.$$

7. Suppose $\underline{u}$ and $\underline{v}$ are non-zero vectors with $|\underline{u} \times \underline{v}| = -\underline{u} \cdot \underline{v}$. Is this possible? If not, why not? If so, determine the angle θ between $\underline{u}$ and $\underline{v}$.

8. Suppose $\underline{u}, \underline{v}, \underline{w}$ is a right-handed triple of vectors. Exactly which of the following are right-handed triples?

(a) $\underline{v} \times \underline{w}, \underline{w}, \underline{v}$

(b) $\underline{w}, \underline{v}, \underline{u}$

(c) $-\underline{v}, \underline{u}, -\underline{w}$

(d) $\underline{u}, -\underline{w}, \underline{v}$

(e) $\underline{v}, \underline{v} \times (-\underline{w}), \underline{w}$

9. Determine the volume of a parallepiped with sides corresponding to $\underline{u} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, and $\underline{w} = \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$.

10. Consider the planes defined by $x - 2y + 2z = 3$ and by $y + z = 0$. Determine a non-zero vector parallel to both of these planes.