Geometry I, 2006 : Mid-term test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $A = (1, 2, 3)$, $B = (2, -1, 4)$. Determine the vector represented by $\overrightarrow{AB}$.

2. Let $A = (1, 2, 3)$, $B = (2, -1, 4)$. Determine the position vector of the point P on the line segment AB, such that $|AP| = \frac{1}{2}|AB|$.

3. Let $A = (1, 2, 3)$, $B = (2, -1, 4)$. Determine a parametric equation of the line through A and B.

4. Let $A = (1, 2, 3)$, $B = (2, -1, 4)$, $D = (2, 0, -3)$. Determine the point C such that $ABCD$ is a parallelogram.

5. Determine the cosine of the angle between the vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$.

6. Determine all solutions of the following system of linear equations in x, y, z:

$$\begin{cases} x - y - 2z = -1 \\ -3x + y + z = 2 \\ 2x - 2y + 4z = -2 \end{cases}.$$

7. Determine the intersection of the plane defined by $x - 2y + 3z = 4$ with the line ℓ through the point $(1, 2, 3)$ and in the direction of $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

8. Determine the vector product $\underline{u} \times \underline{v}$, where $\underline{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$.

9. Exactly which of the following statements are true?

(a) If $\underline{u}$ and $\underline{v}$ are vectors such that $\underline{u} \times \underline{v} = \underline{0}$, then we must have $\underline{u} = \underline{0}$ or $\underline{v} = \underline{0}$.

(b) If $\underline{u}$ is a vector such that $\underline{u} \times \underline{v} = \underline{0}$ for every vector $\underline{v}$, then we must have $\underline{u} = \underline{0}$.

(c) $\underline{u} \times \underline{v} = \underline{v} \times \underline{u}$ for all vectors $\underline{u}, \underline{v}$.

(d) $(\underline{u} \times \underline{v}) \times \underline{w} = \underline{u} \times (\underline{v} \times \underline{w})$ for all vectors $\underline{u}, \underline{v}, \underline{w}$.

(e) $\underline{u} \cdot (\underline{v} \times \underline{w}) = \underline{w} \cdot (\underline{u} \times \underline{v})$ for all vectors $\underline{u}, \underline{v}, \underline{w}$.

10. Let $A = (1, 2, 3)$ and $B = (2, -1, 4)$. Determine a Cartesian equation for a plane through A and B and parallel to the vector $\underline{u} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$.