Queen Mary
University of London

B. Sc. Examination by course unit 2010

MTH4103 Geometry I

Duration: 2 hours

Date and time: 5 May 2010, 1430h–1630h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): L. H. Soicher

Question 1 Let $A = (-3, -1, 2)$ and $B = (1, -2, 3)$. Determine:

(a) the length of the position vector of A; [4]

(b) the vector represented by $\overrightarrow{AB}$; [4]

(c) a vector equation for the line through A and B; [4]

(d) the distance of the point A from the plane with Cartesian equation $3x - 2y + 2z = 4$. [4]

Question 2

(a) Use Gaussian elimination to reduce the following system of linear equations in x, y, z to echelon form. [You are **not** required to solve this system of equations.]

$$\begin{cases} -x & - & 2y & - & z & = & 3 \\ -x & - & y & - & 2z & = & 2 \\ 4x & - & y & - & z & = & -2 \end{cases}.$$

[6]

(b) Apply back substitution to determine **all** solutions of the following system (in echelon form) of linear equations in x, y, z.

$$\begin{cases} -2x & - & 3y & - & z & = & 3 \\ & & & - & 2z & = & 4 \end{cases}.$$

[4]

Question 3 Let $ABCD$ be a parallelogram, with $A = (0, -1, 2)$, $B = (1, 2, 3)$ and $C = (1, 1, -1)$. Determine:

(a) the area of $ABCD$; [6]

(b) a Cartesian equation for the plane through A, B and C. [4]

Question 4 Let $A = \begin{pmatrix} 1 & -1 & 2 \\ -3 & 0 & 1 \\ -2 & 2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -6 & -3 \\ 4 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$.
Determine each of the following:

(a) $\det(A)$; [4]

(b) A^2; [4]

(c) $B - 2C + 3I_2$; [4]

(d) whether B is invertible, and if so, B^{-1}; [4]

(e) all the eigenvalues of B. [4]

Question 5

(a) Define precisely, **not using co-ordinates**, what is meant by the *vector product* $\underline{u} \times \underline{v}$ of vectors $\underline{u}$ and $\underline{v}$. [4]

(b) Apply your definition in part (a) to prove, **without using co-ordinates**, that $\underline{u} \times (\alpha \underline{v}) = \alpha(\underline{u} \times \underline{v})$ for all vectors $\underline{u}, \underline{v}$ and all scalars α. [10]

Question 6 Let S_θ denote the 2×2 matrix representing a reflection (in the (x, y)-plane) in the line through the origin at counterclockwise angle θ (in radians) from the x-axis.

(a) Write down the matrix S_θ, with its entries given explicitly in terms of θ. [4]

(b) Prove that S_θ is invertible, and that $(S_\theta)^{-1} = S_\theta$. [4]

(c) Suppose that A and B are $n \times n$ matrices such that $A^2 = B^2 = (AB)^2 = I_n$. Prove that $AB = BA$. [6]

Question 7 Suppose that $n = 2$ or 3 and that A is an $n \times n$ matrix.

(a) Define what is meant by an *eigenvector* v of A, and what is meant by the *eigenvalue* of A corresponding to v. [4]

(b) Define what is meant by the *characteristic polynomial* of A. [4]

(c) Prove that if λ is an eigenvalue of A and $f(x)$ is the characteristic polynomial of A then $f(\lambda) = 0$. [You may assume, without proof, that if B is an $n \times n$ matrix and zero is an eigenvalue of B then $\det(B) = 0$.] [8]

End of Paper