Queen Mary
University of London

B. Sc. Examination by course unit 2009

MTH4103 Geometry I

Duration: 2 hours

Date and time: 5 May 2009, 1430h–1630h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): L. H. Soicher

Question 1 Let $A = (4, -1, 2)$. Determine:

(a) parametric equations for the line through the point A and in the direction of the vector $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$; [4]

(b) a Cartesian equation for the plane through the point A and orthogonal to the vector $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$; [4]

(c) the vector of length 2 in the same direction as the position vector of A. [4]

Question 2 Consider the vectors $\underline{u} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$. Determine:

(a) the cosine of the angle between $\underline{u}$ and $\underline{v}$; [4]

(b) the area of a parallelogram $ABCD$, such that $\overrightarrow{AB}$ represents $\underline{u}$ and $\overrightarrow{AD}$ represents $\underline{v}$; [4]

(c) the volume of a parallelepiped with sides corresponding to $\underline{u}, -\underline{v}, 3\underline{u}$. [4]

Question 3 Use Gaussian elimination to echelon form, followed by back substitution, to determine **all** solutions of the following system of linear equations in x, y, z:

$$\begin{cases} \quad\;\; y - z = 2 \\ x + 2y + z = 1 \\ 2x + 3y + 3z = 0 \end{cases}.$$

[10]

Question 4 Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ -1 & 2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$. Determine each of the following:

(a) $-A + 2I_3$; [4]

(b) A^2; [4]

(c) the characteristic polynomial of B; [4]

(d) all the eigenvectors of B with corresponding eigenvalue 5. [4]

Question 5 For each of the following linear transformations, determine the 2×2 matrix representing that transformation (you should simplify the matrix entries as much as possible):

(a) the reflection in the (x, y)-plane in the line through the origin at counterclockwise angle $\pi/4$ from the x-axis; [4]

(b) the rotation in the (x, y)-plane, about the origin, through a counterclockwise angle of $3\pi/2$. [4]

Question 6 (a) Define precisely, **not using co-ordinates**, what is meant by the *scalar product* $\underline{u} \cdot \underline{v}$ of vectors $\underline{u}$ and $\underline{v}$. [4]

(b) Suppose now that $\underline{u}$ and $\underline{v}$ are non-zero vectors. Define precisely what it means to say that $\underline{u}$ is *parallel to* $\underline{v}$. [4]

(c) Prove that if $\underline{u}$ and $\underline{v}$ are non-zero vectors and $\underline{u}$ is **not** parallel to $\underline{v}$ then

$$|\underline{u} \cdot \underline{v}| < |\underline{u}||\underline{v}|.$$

[8]

Question 7 (a) Define precisely what it means for a function $t : \mathbb{R}^n \to \mathbb{R}^m$ to be a *linear transformation*. [4]

(b) Apply your definition in part (a) to prove, **without using matrices**, that if $t : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation then $t(0_n) = 0_m$. [4]

(c) Now consider the function $f : \mathbb{R}^n \to \mathbb{R}^m$ defined by $f(u) = 0_m$ for all $u \in \mathbb{R}^n$. Prove that f is a linear transformation. [4]

Question 8 (a) Define precisely, **not using determinants**, what it means for an $n \times n$ matrix A to be *invertible*. [4]

(b) Suppose that A is an invertible $n \times n$ matrix having -1 as one of its eigenvalues. Prove that A^{-1} also has -1 as one of its eigenvalues. [10]

End of Paper