MTH4103 (2013–14)

Queen Mary University of London

Geometry I

Coursework 9 **To hand in on 19th March 2014**

A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 11 Exercise Class.
By the beginning of your Week 11 Exercise Class (on 19th March 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question. Show clearly all the steps in your calculations.

Practice Question 1. Let $s : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with $s\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\3\\-2\end{pmatrix}$, $s\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\4\\-1\end{pmatrix}$ and $s\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}5\\0\\1\end{pmatrix}$, and let $t : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with $t\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\-2\end{pmatrix}$ and $t\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\5\\4\end{pmatrix}$.

(a) Determine matrices representing s, t and $s \circ t$.

(b) Determine $s\begin{pmatrix}-1\\2\\1\end{pmatrix}$.

(c) Determine $(s \circ t)\begin{pmatrix}-3\\4\end{pmatrix}$.

(d) Does $t \circ s$ make sense? Why or why not?

Practice Question 2. For each of the following linear transformations of $\mathbb{R}^2$, determine the 2×2 matrix representing that transformation (simplify your matrix entries as much as possible):

(a) the reflexion in the line $y = -x$;

(b) the rotation about the origin through an angle of $\pi/2$ (anticlockwise);

(c) the transformation made by performing a reflexion in the line $y = -x$ followed by a rotation about the origin through an angle of $\pi/2$ (anticlockwise);

(d) the transformation made by performing a rotation about the origin through an angle of $\pi/2$ followed by a reflexion in the line $y = -x$.

Feedback Question. Hand in your answers to Parts (a), (b), (c), (d) and (f) only.

(a) Define precisely what it means for a function $t : \mathbb{R}^n \to \mathbb{R}^m$ to be a *linear transformation*. [Look in your lecture notes!]

(b) Is the function $t : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $t \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4b \\ 3a + b \end{pmatrix}$ a linear transformation? Why or why not?

(c) Is the function $t : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $t \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a \\ b^3 \end{pmatrix}$ a linear transformation? Why or why not?

(d) Is the function $t : \mathbb{R}^3 \to \mathbb{R}$ defined by $t : \mathbf{r} \mapsto |\mathbf{r}|$ a linear transformation? Why or why not?

(e) Is the function $t : \mathbb{R}^3 \to \mathbb{R}$ defined by $t : \mathbf{r} \mapsto \mathbf{r} \cdot \mathbf{r}$ a linear transformation? Why or why not?

(f) Is the function $t : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $t : \mathbf{r} \mapsto 2(\mathbf{i} \cdot \mathbf{r})\mathbf{k} - 3(\mathbf{r} \times \mathbf{j})$ a linear transformation? Why or why not?

(g) Is the function $t : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $t : \mathbf{r} \mapsto \mathbf{r} \times (2\mathbf{r}) + (\mathbf{i} \cdot \mathbf{r})\mathbf{r} + \mathbf{j} + (\mathbf{k} \times \mathbf{r}) \times \mathbf{r}$ a linear transformation? Why or why not?

Dr John N. Bray, 10th March 2014