MTH4103 (2013–14)

Geometry I

Coursework 5 — To hand in on 12th February 2014

A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 6 Exercise Class.
By the beginning of your Week 6 Exercise Class (on 12th February 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question.
Show clearly all the steps in your calculations. Perform Gaußian elimination and back substitution exactly as specified in your lecture notes.

Practice Question 1.

(a) Determine the intersection of the plane Π defined by $-6x+y+4z=7$ with the line ℓ through the points $(2,3,1)$ and $(1,5,-1)$.

(b) Determine the intersection of the line ℓ through the point $(-3,1,1)$ in the direction of the vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ with the line $\mathfrak{m}$ through the point $(5,-6,0)$ in the direction of the vector $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$.

Practice Question 2. Simplify the following expressions (where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ is a right-handed triple of pairwise orthogonal unit vectors):
(a) $(\mathbf{k}\times\mathbf{i})\times\mathbf{i}$; (b) $\mathbf{k}\times(\mathbf{i}\times\mathbf{i})$; (c) $(3\mathbf{i}\times\mathbf{k})\times(\mathbf{j}\times(-4\mathbf{i}))$; (d) $(4\mathbf{k}\times\mathbf{i})\cdot\mathbf{j}$.

Practice Question 3. Prove (without using coördinates) that

$$\mathbf{u}\times(\alpha\mathbf{v}) = \alpha(\mathbf{u}\times\mathbf{v})$$

for all vectors $\mathbf{u}$, $\mathbf{v}$ and all scalars α. [Hint: Read and understand the proof in the lecture notes that $(\alpha\mathbf{u})\times\mathbf{v} = \alpha(\mathbf{u}\times\mathbf{v})$, and then modify this proof carefully and appropriately.]

Practice Question 4. Let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$.

(a) Determine $\mathbf{v} \times \mathbf{w}$, and then find the volume of a parallelepiped with sides corresponding to $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

(b) Are $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ coplanar? If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are not coplanar then are they a right-handed triple or a left-handed triple? Justify your answers.

Practice Question 5. Suppose $\mathbf{a}$ is a vector with the property that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ for **every** vector $\mathbf{b}$. Prove that we must have $\mathbf{a} = \mathbf{0}$. What can you say about a vector $\mathbf{a}$ having the property that $\mathbf{b} \times \mathbf{a} = \mathbf{0}$ for every vector $\mathbf{b}$?

Feedback Question. For each of the following statements, determine whether it is true or false. If the statement is true then prove it; if it is false then give an example where the statement fails to hold.

(a) If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ is a right-handed triple of vectors, with $\mathbf{u}$ orthogonal to $\mathbf{v}$, then $\mathbf{v}$ must be orthogonal to $\mathbf{w}$.

(b) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times (\mathbf{v} \times \mathbf{v})$ for all vectors $\mathbf{u}$, $\mathbf{v}$.

(c) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{u} = \mathbf{u} \times (\mathbf{v} \times \mathbf{u})$ for all vectors $\mathbf{u}$, $\mathbf{v}$.

(d) If $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$ then we must have $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

Dr John N. Bray, 3rd February 2014