MTH4103 (2013–14)

Geometry I

Coursework 4 **To hand in on 5th February 2014**

A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 5 Exercise Class.
By the beginning of your Week 5 Exercise Class (on 5th February 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question. Show clearly the steps in your calculations. Perform Gaußian elimination and back substitution exactly as specified in your lecture notes.

Practice Question 1. In this practice question, we shall prove together that the elementary operation of adding a multiple of one equation to another does not change the set of solutions of a system of linear equations in x, y, z. (Note that this is obvious for the elementary operation of interchanging two equations, and still fairly obvious for the case of multiplying one of the equations by a nonzero scalar.)
Let $a, b, c, d, e, f, g, h, \alpha$ be real numbers, and consider the following two systems of linear equations:

$$\left.\begin{array}{r} ax + by + cz = d \\ ex + fy + gz = h \end{array}\right\} \quad (*);$$

$$\left.\begin{array}{ccccccc} ax & + & by & + & cz & = & d \\ (e+\alpha a)x & + & (f+\alpha b)y & + & (g+\alpha c)z & = & h+\alpha d \end{array}\right\} \quad (**).$$

We shall prove that these two systems have exactly the same set of solutions, by proving the following two statements.

(a) If $x = p, y = q, z = r$ is a solution of $(*)$ then it is also a solution of $(**)$.

(b) If $x = p, y = q, z = r$ is a solution of $(**)$ then it is also a solution of $(*)$.

I shall prove (a) for you, and then you should prove (b) as an exercise.
Proof of (a): Suppose $x = p, y = q, z = r$ is a solution of $(*)$. This means that $ap + bq + cr = d$ and $ep + fq + gr = h$. In particular, $x = p, y = q, z = r$ is a solution of the first equation of $(**)$. What about the second equation of $(**)$? We have

$$(e + \alpha a)p + (f + \alpha b)q + (g + \alpha c)r = (ep + fq + gr) + \alpha(ap + bq + cr) = h + \alpha d,$$

and so $x = p, y = q, z = r$ is a solution to the second equation of $(**)$ as well as the first, and we are done.

Practice Question 2. For each of the following systems of non-degenerate linear equations in x, y, z in echelon form, use back substitution to determine all solutions of the system:

(a) $$\left.\begin{aligned} 3x + 2y - 5z &= -3 \\ y - z &= 1 \\ 4z &= 1 \end{aligned}\right\};$$

(b) $$7y + 3z = -4 \;\};$$

Practice Question 3. Let Π be the plane defined by

$$3x - 4y + 7z = 2.$$

For each of the three (non-degenerate) linear equations below, determine the intersection of Π with the plane defined by that equation. In each case, give the intersection as a set of points. If the intersection is a line or a plane then also give equation(s) defining this line or plane.

(a) $-6x + 8y - 14z = 4$;

(b) $-6x + 8y - 14z = -4$;

(c) $-6x + 8y - 11z = 8$.

Feedback Question. For each of the following systems of linear equations in x, y, z (over the reals), first use Gaußian elimination to reduce the system to echelon form, and then determine all solutions using back substitution. Remember to deal appropriately with any degenerate equations which may occur.

(a) $$\left.\begin{aligned} -3x - y - 3z &= 2 \\ 4x - 2y - 3z &= -5 \end{aligned}\right\};$$

(b) $$\left.\begin{aligned} 4y - 7z &= 24 \\ 2x - 2y + z &= -8 \\ x + 3y - 3z &= 13 \\ -5x + 3y + 3z &= 4 \end{aligned}\right\}.$$

Dr John N. Bray, 27th January 2014