MTH4103 (2013–14)

Geometry I

Coursework 3 — To hand in on 29th January 2014

Read each question carefully before you start. Use your lecture notes to make sure you understand all terms used in the question and what the question is asking you to do.
A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 4 Exercise Class.
By the beginning of your Week 4 Exercise Class (on 29th January 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question.

Practice Question 1. Let $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$. Determine:

(a) $\mathbf{a}\cdot\mathbf{b}$;

(b) $\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$;

(c) $(\mathbf{a}\cdot(3\mathbf{a}+\mathbf{b}))\mathbf{b}$.

Practice Question 2. Apply the definition of the scalar product contained in your lecture notes to prove that

$$|\mathbf{u}\cdot\mathbf{v}| \leqslant |\mathbf{u}||\mathbf{v}|$$

for all vectors $\mathbf{u}$ and $\mathbf{v}$.

Practice Question 3. Apply the theorem in your your lecture notes which gives a formula for the scalar product of two vectors given in coördinates to prove the following statements.

(a) $(\mathbf{u}+\mathbf{v})\cdot\mathbf{w} = \mathbf{u}\cdot\mathbf{w} + \mathbf{v}\cdot\mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

(b) $(\alpha\mathbf{u})\cdot\mathbf{v} = \alpha(\mathbf{u}\cdot\mathbf{v})$ for all vectors $\mathbf{u}$ and $\mathbf{v}$ and all scalars α.

Feedback Question. Show your calculations and indicate clearly what you are calculating.

(a) Let Π be the plane through the point $(1, -3, -7)$ and orthogonal to the vector $\begin{pmatrix} -2 \\ 5 \\ -6 \end{pmatrix}$. Determine a Cartesian equation for Π.

(b) Is the point $(6, 1, 3)$ on Π? Why or why not?

(c) Is the point $(-6, -1, -3)$ on Π? Why or why not?

(d) Determine the distance of the point $(3, -1, 3)$ from Π.

(e) Determine the distance of the point $(0, -1, -5)$ from Π.

(f) In this question I shall abuse language, and use "$\mathbf{a}$ is on Π" to mean "the point A with position vector $\mathbf{a}$ is on Π". Let Π be a plane containing a vector $\mathbf{u}$ such that $-\mathbf{u}$ is also on Π. What is the most general form of a possible Cartesian equation for Π? Deduce that for such a plane $-\mathbf{u}$ is on Π for all $\mathbf{u}$ on Π.

Dr John N. Bray, 20th January 2014