MTH4103 (2013–14)

Queen Mary University of London

Geometry I

Coursework 2 — To hand in on 22nd January 2014

Read each question carefully before you start. Use your lecture notes to make sure you understand all terms used in the question and what the question is asking you to do.
A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 3 Exercise Class.
By the beginning of your Week 3 Exercise Class (on 22nd January 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question.

Practice Question 1. Let A, B, C, D be **any** four points in 3-space, and let P, Q, R, S be the respective mid-points of the line segments AB, BC, CD, DA.

(a) Apply Theorem 1.11 in your lecture notes to determine the position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$, $\mathbf{s}$ of P, Q, R, S in terms of the position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ of A, B, C, D.

(b) Using these position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$, $\mathbf{s}$, determine expressions in $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ for the vector represented by $\overrightarrow{PQ}$ and for the vector represented by $\overrightarrow{SR}$.

(c) Conclude that $PQRS$ must be a parallelogram (even though the figure $ABCD$ we started with certainly need not be one).

Practice Question 2. Let $\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$. Determine:

(a) $3\mathbf{a} - 5\mathbf{b}$;

(b) $|\mathbf{a} + 2\mathbf{b}|$;

(c) the vector of length 1 in the same direction as $\mathbf{a}$;

(d) the vector of length 1 in the opposite direction to $\mathbf{a}$;

(e) the vector of length 3 in the same direction as $\mathbf{b}$.

Practice Question 3.

(a) Determine a vector equation, parametric equations, and Cartesian equations for the line through the point $(3, -1, 3)$ in the direction of the vector $\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$.

(b) Is the point $(-1, -1, -7)$ on this line? Why or why not?

(c) Is the point $(2, -1, 7)$ on this line? Why or why not?

Practice Question 4. Let $A = (-2, 3, 5)$ and $B = (2, 4, 7)$, with respective position vectors $\mathbf{a}$ and $\mathbf{b}$, and let ℓ be the line through the points A and B.

(a) Determine (in coördinates) $\mathbf{a}$, $\mathbf{b}$ and the vector represented by $\overrightarrow{AB}$.

(b) Determine a vector equation, parametric equations, and Cartesian equations for the line ℓ.

(c) Determine the point C such that $\overrightarrow{BC}$ represents $\mathbf{b} - \mathbf{a}$. Is C on ℓ? Why or why not?

Feedback Question.

(a) Define (in terms of their scalar product) what it means for vectors $\mathbf{a}$ and $\mathbf{b}$ to be *orthogonal*.

(b) Let $\mathbf{a} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$. Determine a nonzero vector orthogonal to $\mathbf{a}$.

(c) Now suppose $\mathbf{a}$ is a vector with the property that $\mathbf{a} \cdot \mathbf{b} = 0$ for **every** vector $\mathbf{b}$. Prove that we must have $\mathbf{a} = \mathbf{0}$.

(d) Prove that there is **no** vector $\mathbf{a}$ with the property that $\mathbf{a} \cdot \mathbf{b} = 1$ for **every** vector $\mathbf{b}$.

Dr John N. Bray, 13th January 2014