MTH4103 (2013–14)

Geometry I

Coursework 1 — To hand in on 15th January 2014

Read each question carefully before you start. Use your lecture notes to make sure you understand all terms used in the question and what the question is asking you to do.
A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 2 Exercise Class.
By the beginning of your Week 2 Exercise Class (on 15th January 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question.

Practice Question 1. Prove (without using coördinates) that $\mathbf{u}+\mathbf{0}=\mathbf{u}$ for each vector $\mathbf{u}$. [Hint: Let $\overrightarrow{AB}$ represent $\mathbf{u}$, and then represent $\mathbf{0}$ appropriately to be able to apply the Triangle Rule.]

Practice Question 2. Prove, using theorems in the notes (Theorems 1.2, 1.4, 1.5 and 1.6), that for all vectors $\mathbf{u}$ and $\mathbf{v}$ we have $((-\mathbf{u})+\mathbf{v})+\mathbf{u}=\mathbf{v}$. (Recall that these theorems state various basic properties of vector addition.)

Practice Question 3. In lectures we showed that $(-1)\mathbf{v}=-\mathbf{v}$ and $\alpha(\beta\mathbf{v})=(\alpha\beta)\mathbf{v}$ for all scalars α, β and all vectors $\mathbf{v}$. Apply these facts to deduce (without using coördinates) that:

$$-(\lambda\mathbf{w})=(-\lambda)\mathbf{w}=\lambda(-\mathbf{w})$$

for all scalars λ and all vectors $\mathbf{w}$. Note that there are two equalities to prove. You should show clearly each step used in your deductions.

Feedback Question. Let O be a fixed origin in 3-space, let P and Q be any points in this space, with respective position vectors $\mathbf{p}$ and $\mathbf{q}$, and let R be the point such that $OPQR$ is a parallelogram.

(a) Define precisely what it means to say that the figure $OPQR$ is a *parallelogram*. [Hint: Consult your lecture notes.]

(b) Define precisely what is meant by the *position vector* $\mathbf{p}$ of P.

(c) For each of the following bound vectors, determine an expression in terms of $\mathbf{p}$ and $\mathbf{q}$ for the free vector represented by that bound vector:

(i) $\overrightarrow{OP}$; (ii) $\overrightarrow{QO}$; (iii) $\overrightarrow{OR}$; (iv) $\overrightarrow{PQ}$; (v) $\overrightarrow{QR}$; (vi) $\overrightarrow{RP}$.

In each case explain your reasoning. [Hint: Remember that in this question, $OPQR$ is a parallelogram. Draw a diagram to help you think.]

Dr John N. Bray, 6th January 2014