Queen Mary
University of London

MAE113 **Discrete Techniques for Computing**

In-term Test **Wednesday 10 November 2010, 12:10-12:50**

Write your name and student number in the spaces below.

Answer all questions. Write all your answers in the boxes provided[1] *and show all your working.*

Electronic calculators may not be used in this examination.

Do not start reading the paper until instructed by the invigilators.

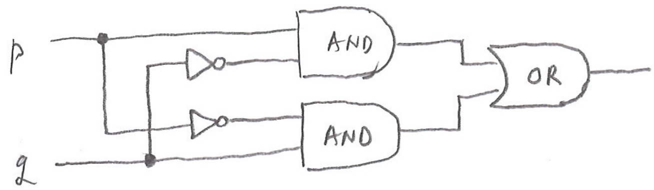
Name: ____________________________________

Student Number: ____________________________

[1] If more space is needed, write on the blank pages.

1 (25 marks)

(a) Draw the logic circuit given by the formula $(p \wedge \neg q) \vee (\neg p \wedge q)$. Write out its truth table.



[7 MARKS]

p	q	$(p \wedge \neg q) \vee (\neg p \wedge q)$				
1	1	0	0	0	0	0
1	0	1	1	1	0	0
0	1	0	0	1	1	1
0	0	0	1	0	1	0

[8 MARKS]

(b) Write the Boolean formula $(p \to q) \to r$ as a disjunction of minterms and simplify the result as much as possible.

Write the truth table, read off the DNF, simplify!

p	q	r	$(p \to q)$	$\to r$
1	1	1	1	1
1	1	0	1	0
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	1	0
0	0	1	1	1
0	0	0	1	0

[3 MARKS]

DNF: [4 MARKS]

$$pqr \vee pq'r \vee pq'r' \vee p'qr \vee p'q'r$$

$$\equiv pr \vee pq'r' \vee p'r$$

$$\equiv r \vee pq'r'$$

[3 MARKS]

Simplifying in a different way, another acceptable answer is $pqr \vee pq' \vee p'r$

2 (25 marks)

(a) Carry out the binary multiplication 10110×1011.

10110 · 1011

10110   1011
10110   101
~~10110~~   10
10110   1

11110010

[6 MARKS]

[7 MARKS]

(b) Carry out the binary subtraction $100001 - 10111$.

$$
\begin{array}{r}
100001 \\
-\ 10111 \\
\hline
001010
\end{array}
$$

[12 MARKS]

CHECK:

$$
\begin{array}{r}
10111 \\
+\ 1010 \\
\hline
100001
\end{array}
$$

3 (25 marks) Suppose we are given sets A, B, C such that $|A| = 16$, $|B| = 16$, $|C| = 18$, $|A \cap B| = 10$, $|A \cap C| = 11$, $|B \cap C| = 8$ and $|A \cap B \cap C| = 7$.

(a) Calculate $|A \cup B|$.

$$|A \cup B| = |A| + |B| - |A \cap B| = 16 + 16 - 10 = 22$$

[12 MARKS]

(b) Calculate $|A \cup B \cup C|$ using the principle of inclusion-exclusion.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 16 + 16 + 18 - 10 - 11 - 8 + 7$$

$$= 28$$

[13 MARKS]

4 (25 marks) Let us work in $\mathbb{Z}_9 = \{[0],[1],[2],[3],[4],[5],[6],[7],[8]\}$.

(a) Calculate $([5]+[7])\cdot([2]-[6])$ in $\mathbb{Z}_9$.

$$([5]+[7])\cdot([2]-[6]) = [3]\cdot[-4] = [3]\cdot[5] = [15] = [6]$$

[10 MARKS]

(b) Does there exist an element $[x]$ in $\mathbb{Z}_9$ such that $[4]\cdot[x] = [1]$? Find it if it does!

$[x]$	0	1	2	3	4	5	6	7	8
$[4]\cdot[x]$	0	4	8	3	7	2	6	1	5

YES, $[x] = [7]$.

[8 MARKS]

(c) Does there exist an element $[y]$ in $\mathbb{Z}_9$ such that $[3]\cdot[y] = [1]$? Find it if it does!

$[y]$	0	1	2	3	4	5	6	7	8
$[3]\cdot[y]$	0	3	6	0	3	6	0	3	6

1 does not appear!

THERE IS NO SUCH $[y]$!

[7 MARKS]