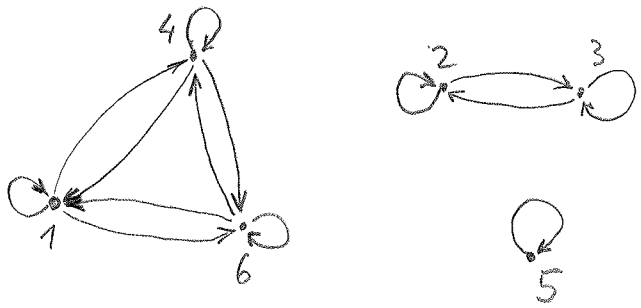
CW6 - SOLUTIONS

(1)



(i) It is REFLEXIVE since there is a loop at each point.

(ii) It is SYMMETRIC since whenever we have  there is also an arrow 

(iii) It is TRANSITIVE since any two points which are connected by a chain of arrows are also directly connected.

(2) (a) NOT REFLEXIVE SINCE e.g. $1 \not R 1$, i.e. $(1,1) \notin R$

SYMMETRIC since whenever aRb, we also have bRa

NOT TRANSITIVE : $1R2$, $2R1$ but $1 \not R 1$.

(b) REFLEXIVE since for any $a \in A$, $a \leq a$.

NOT SYMMETRIC : $1 \leq 2$ BUT $2 \not\leq 1$

TRANSITIVE : $a \leq b$ & $b \leq c$ implies $a \leq c$.

(c) REFLEXIVE : for any $a \in \mathbb{Z}$, aRa since $a+a = 2a$ is even.

SYMMETRIC : for any $a, b \in \mathbb{Z}$, if $aRb \Rightarrow a+b$ is even

$\Rightarrow b+a$ is even $\Rightarrow bRa$

TRANSITIVE : if aRb AND $bRc \Rightarrow$ $a+b = 2 \cdot n$, $b+c = 2 \cdot m$ $\Big| +$ for some $m, n \in \mathbb{Z}$.

$a+2b+c = 2(n+m)$

$a+c = 2(n+m-b)$ ← EVEN

$\Rightarrow aRc$

(3) (a) In $\mathbb{Z}_{17}$, $[14]+[5]=[19]=[2]$

(b) In $\mathbb{Z}_{23}$, $[12]-[19]=[-7]=[-7+23]=[16]$

(c) In $\mathbb{Z}_{21}$, $[11]\cdot[18]\cdot[19]\cdot[6]\cdot[17]=[11]\cdot[-3]\cdot[-2]\cdot[6]\cdot[-4]$

$=[11][6]\cdot[-24]=[11]\cdot[6][-3]=[11]\cdot[-18]=[11]\cdot[3]$

$=[33]=[12]$

(d) In $\mathbb{Z}_{36}$, $\big([22]+[17]\big)\cdot\big([1]-[33]\big)=[3]\cdot[-32]=[3]\cdot[4]=[12]$

(4) (a) We need to find $x\in\mathbb{Z}_{11}=\{[0],[1],[2],\ldots,[10]\}$ s.t.

$x\cdot[8]=[3]$.

By trial and error, the only such x is $x=[10]$;

$[10]\cdot[8]=[80]=[3]$.

(b) Is there an x s.t. $x\cdot[2]=[8]$ in $\mathbb{Z}_{10}$?

x	0	1	2	3	4	5	6	7	8	9
$x\cdot[2]$	0	2	4	6	8	0	2	4	6	8

Yes, $x=4$ and $x=9$ satisfy this, but this means that $\frac{[8]}{[2]}$ IS NOT WELL DEFINED in $\mathbb{Z}_{10}$.

(c) By inspection, $\frac{[1]}{[13]}=[7]$, since $[7]\cdot[13]=[91]=[1]$.

(5) Enumerate:

SUN - 0
MON - 1
TUE - 2
WED - 3
THU - 4
FRI - 5
SAT - 6

(a) Working in $\mathbb{Z}_7$, $[1+549]=[550]=[4]$,
so it will be Thursday.

(b) $6126=255\cdot24+6$ so
6126 hours is 255 days and 6 hours

in $\mathbb{Z}_7$, $[1+255]=[256]=[4]$

So it will be 16:30 on a Thursday.