CW2 - SOLUTIONS

① (a) $pqr \vee p'qr' \vee p'q'r' \vee pq'r \equiv (pqr \vee pq'r) \vee (p'qr' \vee p'q'r')$

$\equiv pr \vee p'r'$, which doesn't simplify any further.

(b) $pqr \vee pq'r' \vee pqr' \vee pq'r \equiv (pqr \vee pqr') \vee (pq'r' \vee pq'r)$

$\equiv pq \vee pq' \equiv p$

(c) $pq'rs't \vee p'r'st' \vee pq'r's't \vee p'qr's't' \vee p'q'r's't' \equiv$

$\equiv (pq'rs't \vee pq'r's't) \vee p'r'st' \vee (p'qr's't' \vee p'q'r's't') \equiv$

$\equiv pq's't \vee p'r'st' \vee p'r's't' \equiv pq's't \vee (p'r'st' \vee p'r's't') \equiv$

$\equiv pq's't \vee p'r't'$

② (a) False (c) True (e) True

(b) False (d) False (f) False

③ (I) UNINSPIRED SOLUTION, 100% SAFE:

we shall write out the truth tables, read off the DISJUNCTIVE NORMAL FORM and then simplify it.

p	q	r	$(p \wedge q) \longrightarrow r$		$(p \vee q) \longleftrightarrow r$	
1	1	1	1	1	1	1
1	1	0	1	0	1	0
1	0	1	0	1	1	1
1	0	0	0	1	1	0
0	1	1	0	1	1	1
0	1	0	0	1	1	0
0	0	1	0	1	0	0
0	0	0	0	1	0	1

③ (CONT)

(a) NOTE that the DNF for $(p \wedge q) \to r$ would be quite big,

but $\neg((p \wedge q) \to r)$ has all zeros except in row $\begin{array}{ccc} p & q & r \\ 1 & 1 & 0 \end{array}$.

Thus $\neg((p \wedge q) \to r) \equiv p q r'$.

$$\Rightarrow \quad (p \wedge q) \to r \equiv \neg(p q r') \overset{\text{DE MORGAN}}{\equiv} p' \vee q' \vee r$$

(b) DNF for $(p \vee q) \to r$ is:

$$(p \vee q) \to r \equiv p q r \vee p q' r \vee p' q r \vee p' q' r' \equiv p q r \vee p q r \vee p q' r \vee p' q r \vee p' q' r'$$

$$\equiv q r \vee p r \vee p' q' r' \quad \leftarrow \text{This is the simplest DNF,}$$

using the DISTRIBUTIVE LAW it simplifies to

$$\equiv (p \vee q) r \vee p' q' r' \underset{\text{DE MORGAN}}{\equiv} (p \vee q) r \vee (p \vee q)' r'$$

(II) A MORE IMAGINATIVE SOLUTION:

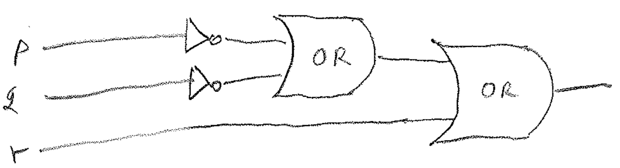
Using the fact that $A \to B \equiv A' \vee B$, $A \leftrightarrow B \equiv AB \vee A'B'$

(a) $$pq \to r \equiv (pq)' \vee r \underset{\text{DE MORGAN}}{\equiv} p' \vee q' \vee r$$

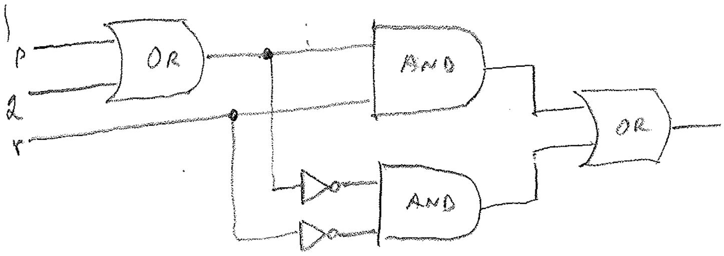
(b) $$(p \vee q) \leftrightarrow r \equiv (p \vee q) r \vee (p \vee q)' r'$$

CIRCUITS:

(a)



(b)



4 (a)

p	$p \wedge p'$	
1	0	0
0	0	1

Always false, so INCONSISTENT

(b), (c)

p	q	$(p \to q) \leftrightarrow (p' \vee q)$				$((p' \to q) \wedge (p' \to q')) \to p$				
1	1	1	1	0	1	1	1	1	0	1
1	0	0	1	0	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	1
0	0	1	1	1	1	0	0	1	1	1

(b) is always true, so TAUTOLOGY

(c) is a TAUTOLOGY

FOR QUESTION 5(a)

(d)

p	q	r	$((p \wedge q) \to r) \leftrightarrow (p \to (q \to r))$					$(p \to r) \vee (q \to r)$		
1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	0	0	0	0
1	0	1	0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	0	1	1	0
0	0	1	0	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1	1

TAUTOLOGY

5 (a)

SAME OUTPUTS, so $(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$

(b) $(p \wedge q) \to r \equiv (p \wedge q)' \vee r \equiv p' \vee q' \vee r \equiv p' \vee q' \vee r \vee r$

$\equiv (p' \vee r) \vee (q' \vee r) \equiv (p \to r) \vee (q \to r)$