Queen Mary, University of London

MAE113 Discrete Techniques for Computing, 2007. Solutions to SAMPLE MID-TERM TEST.

1. (a) $X \cap Y = \{0, 3\}$. [3 marks]

   $X \cup Y = \{0, 1, 2, 3, 6, 9\}$, so $|X \cup Y| = 6$. [5 marks]

   (OR $|X \cup Y| = |X| + |Y| - |X \cap Y| = 4 + 4 - 2 = 6$)

   (b) By the principle of inclusion- exclusion,

   $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

   $= 100 + 30 + 34 - 6 - 23 - 8 + 3 = 130$. [12 marks]

2. The boolean formula of the circuit is $(pq)'q$. [8 marks]

p	q	pq	$(pq)'$	$(pq)'q$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	T
F	F	F	T	F
				$\Uparrow$

[12 marks]

3. (a) $pqrs \vee p'qrs' \vee pq'rs \vee p'qr's' \equiv (pqrs \vee pq'rs) \vee (p'qrs' \vee p'qr's')$

   $\equiv prs \vee p'qs'$. [8 marks]

   (b)

p	q	$p \leftrightarrow q$	q'	$q'p$	$(p \leftrightarrow q) \vee (\neg q \wedge p)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	T	T	F	T
					$\Uparrow$

[8 marks]

Equivalent to $pq \vee pq' \vee p'q'$. [4 marks]

(OR $p \vee p'q'$ OR $pq \vee q'$ would be correct.)

4. (a)

1	431
1	215
1	107
1	53
0	26
1	13
0	6
1	3
1	1

So 431 (in the decimal system) is 110101111 in the binary system.

[8 marks]

(b)

$$
\begin{array}{rcr}
1101101 & & \\
\times 10101 & & \\
\hline
1101101 & 1 & 1101101 \\
 & 0 & +110110100 \\
\hline
110110100 & 1 & 1000100001 \\
 & 0 & +11011010000 \\
\hline
11011010000 & 1 & 100011110001
\end{array}
$$

So $1101101 \times 10101 = 100011110001$. [12 marks]

5. In $\mathbb{Z}_{11} = \{[0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]\}$.

(a)
$[5] + ([2] \times [6]) + [8]([1] - [5]) = [5] + [1] + [8]([1] + [6]) = [6] + [1] = [7]$.

[8 marks]

(b) $[3][9] = [27] = [5]$, so $n = 9$. [12 marks]