

Queen Mary, University of London
MAE113 DISCRETE TECHNIQUES FOR
COMPUTING

Mid-Term Test, November 12th 2009, 12pm.

Time allowed: 45 minutes

Each question carries 20 marks, making a total of 80. Write your answers clearly and show all your working.

1. (a) Let A be the set $\{1, 2, 3, 4\}$ and let B be the set consisting of all those numbers which are equal to twice some number in A . Calculate $A \cap B$ and $A \cup B$.
(b) Suppose A , B and C are sets, and we are given that $|A| = 45$, $|A \cap B| = 21$, $|A \cap C| = 20$, $|A \cap B \cap C| = 9$ and $|B \cup C| = 65$. Calculate $|A \cup B \cup C|$ using the inclusion-exclusion principle.
2. (a) Carry out the binary multiplication 10101×1010 . Check your answer by converting the binary numbers to decimal.
(b) Carry out the binary subtraction $10101 - 1010$.
3. \mathbb{Z}_8 consists of the equivalence classes $[0], [1], [2], [3], [4], [5], [6], [7]$.
(a) For which equivalence classes $[x]$ in \mathbb{Z}_8 can we find a $[y]$ such that $[x] \times [y] = [1]$? Explain your answer.
(b) Calculate in \mathbb{Z}_8 :
 - (i) $([2] + [7]) \times ([1] - [6])$,
 - (ii) $[3] \div [5]$.
4. (a) Draw a logic circuit whose output is given by the formula $pq' \vee p' \vee pr$. Also write out its truth table.
(b) Find a Boolean formula which is equivalent to $(p' \vee q) \rightarrow r$ and is a disjunction of at most three minterms.