MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Coursework 6–to be handed in by noon, Wednesday 17/11/2010.

Write your name and student number at the top of your assignment before handing it in. You should attempt all questions because only one question will be marked.

1. Let A be the set $\{1, 2, 3, 4, 5, 6\}$ and let R be the binary relation

   $$\{(1, 1), (1, 4), (1, 6), (2, 2), (2, 3), (3, 2), (3, 3), (4, 1), (4, 4), (4, 6), (5, 5), (6, 1), (6, 4), (6, 6)\}.$$

   Draw the graph of R. Is R: (i) Reflexive, (ii) Symmetric, (iii) Transitive? How can we recognise these properties from the graph of R?

2. For each of the following sets and binary relations, state whether or not they are (i) Reflexive, (ii) Symmetric, (iii) Transitive:

   (a) $A := \{1, 2, 3, 4\}$, $R := \{(1, 2), (1, 3), (2, 3), (2, 1), (3, 1), (3, 2), (1, 4), (4, 1))\}$,

   (b) $A := \{1, 2, 3, 4, 5\}$, $R := \{(a, b) : a \in A, b \in A, a \leq b\}$,

   (c) $A := \mathbb{Z}$, $R := \{(a, b) : a \in A, b \in A, a + b \text{ is an even number}\}$.

   Justify your answers.

3. Calculate:

   (a) $[14] + [5]$ in $\mathbb{Z}_{17}$,

   (b) $[12] - [19]$ in $\mathbb{Z}_{23}$,

   (c) $[11] \cdot [18] \cdot [19] \cdot [6] \cdot [17]$ in $\mathbb{Z}_{21}$ (a calculator is not necessary),

   (d) $([22] + [17]) \cdot ([1] - [33])$ in $\mathbb{Z}_{36}$ (again, no calculator required).

4. Calculate, or else explain why no answer exists:

   (a) $[3] \div [8]$ in $\mathbb{Z}_{11}$.

   (b) $[8] \div [2]$ in $\mathbb{Z}_{10}$.

   (c) $[1] \div [13]$ in $\mathbb{Z}_{15}$.

5. (Applying modular arithmetic in real life.) Suppose the time is 10:30am on a Monday.

   (a) Which day of the week will be in 549 days from that time?

   (b) What will be the time and day of the week in 6126 hours from the given time?