MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Coursework 2–to be handed in by noon, Wednesday 13/10/2010.

Write your name and student number at the top of your assignment before handing it in. You should attempt all questions because only one question will be marked.

1. Simplify the following Boolean expressions as much as possible:
   (a) $pqr \vee p'qr' \vee p'q'r' \vee pq'r$,
   (b) $pqr \vee pq'r' \vee pqr' \vee pq'r$,
   (c) $pq'rs't \vee p'r'st' \vee pq'r's't \vee p'qr's't' \vee p'q'r's't'$.

2. Mark the following propositions true or false:
   (a) $(2^2 = 4) \to (2 + 3 > 6)$,
   (b) $(2^2 = 4) \leftrightarrow (2 + 3 > 6)$,
   (c) (London is the capital of France)$\to (2 + 3 = 5)$,
   (d) (Paris is the capital of England)$\leftrightarrow (2 + 3 = 5)$,
   (e) $(3 \times 4 = 12) \to$ (London is the capital of England),
   (f) $((10 \div 2 = 5) \to (3 < 5)) \to (4 \leq 1)$.

3. Using only logical gates AND, OR and NOT, design the logical circuit (using the fewest number of gates) whose output is equivalent to the formulae:
   (a) $pq \to r$,
   (b) $(p \vee q) \leftrightarrow r$.

4. A *tautology* is a boolean formula which is always true (no matter which inputs we take). A boolean formula is *inconsistent* if it is always false. Which of the following Boolean expressions are tautologies, which are inconsistent, and which are neither? Justify your answers with truth tables.
   (a) $p \wedge p'$,
   (b) $(p \to q) \leftrightarrow p' \vee q$,
   (c) $((p' \to q) \wedge (p' \to q')) \to p$,
   (d) $(pq \to r) \leftrightarrow (p \to (q \to r))$.

5. Prove the Boolean equivalence $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ in two ways:
   (a) By constructing truth tables,
   (b) Using the laws of Boolean algebra (hint: you can use $(p \to q) \equiv p' \vee q$).