## MTH5118 Probability II. Problem Sheet 8.

This coursework consists of two parts. You are required to submit solutions to the second part only. You are strongly encouraged to solve all problems on this problem sheet. Please staple your coursework and post it in the Blue Box in the basement of the Maths building by 10:30 on Thursday, 3 December 2009.

## Part 1

1. Random variables $X$ and $Y$ have joint p.d.f. $f_{X, Y}(x, y)=\frac{12}{7}\left(x^{2}+x y\right)$ for $0<x<1$, $0<y<1$ and $f_{X, Y}(x, y)=0$ elsewhere. Find $P(X+Y \leq 1)$.
2. In each of the following cases identify if $X$ and $Y$ are independent random variables and justify your result. For the cases where they are independent write down the marginal p.d.f. and range for each of $X$ and $Y$ and state the value of $C$.
(a) $f_{X, Y}(x, y)=C(1+x+x y)$ for $0<x<1$ and $0<y<1$ and the p.d.f. is zero elsewhere.
(b) $f_{X, Y}(x, y)=C e^{-x-3 y}$ for $0<x<\infty$ and $0<y<\infty$ and the p.d.f. is zero elsewhere.
(c) $f_{X, Y}(x, y)=C x^{2}(1+y)$ for $x>0, y>0$ and $x+y<1$ and the p.d.f. is zero elsewhere.
(d) $f_{X, Y}(x, y)=C \frac{x e^{-2 x}}{y^{2}}$ for $0<x<\infty$ and $1<y<\infty$ and the p.d.f. is zero elsewhere.

## Part 2

3. Random variables $X$ and $Y$ have joint p.d.f. $f_{X, Y}(x, y)=2 e^{-(x+y)}$ for $0<x<y<\infty$ and $f_{X, Y}(x, y)=0$ elsewhere. Find $P(Y-X>z)$ for $z>0$. (Note that this probability is one if $z \leq 0$ ).

Let $Z=Y-X$. Use your result to give the c.d.f. for $Z, F_{Z}(z)=P(Z \leq z)=10$ $1-P(Z>z)$. Hence find the p.d.f. of $Z$.
4. Let $X$ and $Y$ be independent chi-squared random variables each with parameter $n$. State their m.g.f.'s, $M_{X}(t)$ and $M_{Y}(t)$.
(a) Let $U=X+Y$. Obtain the m.g.f. of $U$ by writing it in terms of the m.g.f.'s for $X$ and $Y$. Hence state the distribution of $U$.
(b) Let $V=X-Y$. Obtain the m.g.f. of $V$ by writing it in terms of the m.g.f.'s for $X$ and $Y$.

For the case when $n=2$, compare $M_{V}(t)$ with the m.g.f. for a double exponential (see 10 Q4 of Problem Sheet 6). Hence state the p.d.f. of $V$ when $n=2$.
5. Let $X$ and $Y$ have joint p.d.f. $f_{X, Y}(x, y)=(x+y)$ for $0<x<1$ and $0<y<1$ and 20 $f_{X, Y}(x, y)=0$ elsewhere. Find the coefficient of correlation between $X$ and $Y$.

