MTH5118 Probability II. Problem Sheet 6.

This coursework consists of two parts. **You are required to submit solutions to the second part only. However, you are strongly encouraged to solve all problems on this problem sheet.** *Please staple your coursework and post it in the Blue Box in the basement of the Maths building by* **10:30** *on Thursday, 19th November 2009.*

Part 1

1. The probability density function (p.d.f.) of X is given by $f_X(x) = Cx(2-x)$ if $0 < x < 2$ and is zero elsewhere. Find C and the cumulative distribution function (c.d.f.) of X, $F_X(x)$. Obtain the mean, $E[X]$, and show that the median is equal to the mean. (By definition, C is a median if $P(X \leq C) = \frac{1}{2}$.)

2. Let X have m.g.f. $M_X(t) = e^{\alpha t}\left(1 - \frac{t}{\theta}\right)^{-1}$. Differentiate the m.g.f. to obtain $E[X]$ and $Var(X)$.

Part 2

3. X and Y have trinomial distribution with parameters n, p and θ. You may assume that the marginal distributions are $X \sim Binomial(n,p)$ and $Y \sim Binomial(n,\theta)$. Let $Z = X + Y$.

(a) What is the distribution of Z? 15

(b) Show that the conditional distribution of $X|Z = z$ is binomial and specify the parameters. 20

Hint. Recall the definitions of the Binomial and trinomial distributions.

4. X has double exponential distribution with p.d.f. $f_X(x) = \frac{\theta}{2}e^{-\theta|x|}$ for all $-\infty < x < \infty$ (where the parameter $\theta > 0$).

(a) Find the c.d.f. of $Y = |X|$ and hence obtain the p.d.f. and state the distribution of Y. 20

(b) Show that the m.g.f. of X is $M_X(t) = \left(1 - \frac{t^2}{\theta^2}\right)^{-1}$ for $|t| < \theta$. By expanding the m.g.f. in a power series obtain the mean μ, variance σ^2 and the coefficients of skewness and kurtosis $\sqrt{\beta_1} = \frac{E[(X-\mu)^3]}{\sigma^3}$ and $\beta_2 = \frac{E[(X-\mu)^4]}{\sigma^4}$. 25

(c) Let $Z = Y^3$. Find the p.d.f. of Z. 20