MTH5118 Probability II. Problem Sheet 5.

This coursework consists of two parts. **You are required to submit solutions to the second part only. However, you are strongly encouraged to solve all problems on this problem sheet.** *Please staple your coursework and post it in the Blue Box in the basement of the Maths building by* **10:30** *on Thursday, 5th November 2009.*

Part 1

1. X and Y have the joint trinomial distribution with parameters n, p and θ. You may assume that the marginal distributions are $X \sim Binomial(n,p)$ and $Y \sim Binomial(n,\theta)$.

(i) Obtain the conditional distribution of $X|Y=y$ and state $E[X|Y]$. Use the result that $E[XY] = E[Y \times E[X|Y]]$ to obtain $Cov(X,Y)$.

(ii) Let $Z = X+Y$. By writing $G_Z(t)$ in terms of the joint p.g.f. for X and Y, obtain the distribution for Z. Show that the conditional distribution of $X|Z=z$ is binomial and specify the parameters.

Part 2

2.A. (i) Let $S_N = \sum_{j=1}^{N} X_j$, where X_j are i.i.d.r.v.'s with $E(X_j) = a$, $Var(X_j) = \sigma^2$ and N is an integer-valued non-negative r.v. independent of the sequence X_j. Write down the formulae for $E(S_N)$ and $Var(S_N)$ derived in the lecture. 5

(ii) The number of customers arriving at a shop during one day is a r.v. $N \sim Poisson(100)$. The customers act independently of each other spending a random amount of X_j pounds in the shop. Given that $E(X_j) = 20$, $Var(X_j) = 40$, find the average value and the variance of the daily cash flow S in this shop. 15

2.B. The number of spam messages Y in a day has Poisson distribution with parameter μ. Each spam message (independently) has probability p of not being detected by the spam filter. Let X be the number of messages getting through the filter.

(i) Find $E(X)$ and $Var(X)$. (Hint: note that $X = \sum_{j=1}^{Y} Z_j$, where $Z_j \sim Bernoulli(p)$ i.i.d.r.v.'s.) 15

(ii) Compute $E(t^X)$ and hence find the p.g.f. of X. To this end: 1. Compute $E(t^X|Y=y)$; 2. Use the fact that $E(t^X) = E[E(t^X|Y)]$. 25

(iii) Hence find the p.m.f. of X and identify the name of this distribution. 10

(iv) Repeat (i) and (ii) if $Y \sim Geometric(\bar{p})$, that is $P(Y=k) = \bar{p}\bar{q}^k$, $k = 0, 1, 2, \ldots.$ 30