MTH5118 Probability II. Problem Sheet 4.

(d) Obtain the results in (b) and (c) if initially there are 3 bacteria.

This time you are required to submit solutions to all problems on this problem sheet. Please staple your coursework and post it in the Blue Box in the basement of the Maths building by 10:30 on Thursday, 29th October 2009.

1. A population of bacteria begins with a single individual (forming generation 0). In each generation, each individual dies with probability 1/4, doubles (splits in two) with probability 1/2, and triples (splits in 3) with probability 1/4. Let Y_n be the number of bacteria in generation n.

a) Find the probability θ_3 that the population will die out by generation 3.	10
(b) Find $E[Y_3]$ and $Var(Y_3)$.	10
(c) Find the probability θ that the population will eventually die out.	15

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2. The family surname survives through the male line of descent. Each male in a particular society has probability 1/4 of having no children and probability 3/4 of having exactly 2 children. Each child has probability 3/4 of being a boy, independently of the other children. Find the p.g.f., $G_X(t)$, for the number of sons X for a male in the society. 15

N newborns in the society are given the surname of Ramsbottom in honour of a benefactor from abroad. No-one else in the society has that name. Find the probability that the surname Ramsbottom will die out eventually in the society if:

(i) exactly k out of the N newborns selected are boys;	15
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(ii) the number K of newborns selected who are boys is a random variable with $K \sim 25$ Binomial $\left(N, \frac{1}{2}\right)$ (Hint: use the result from (i) and the theorem of total probability.)