MTH5118 Probability II. Problem Sheet 2.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY PLEASE

This coursework consists of two parts. **You are required to submit solutions to the second part only. However, you are strongly encouraged to solve all problems on this problem sheet.** *Please staple your coursework and post it in the Blue Box in the basement of the Maths building by* **10:30** *on Thursday, 15th October 2009.*

Part 1

1. The roulette wheel at a casino has integers from 1 to 36, together with 0. Any of the numbers between 0 and 36 is equally likely to occur each time the wheel is spun.

Kermit has £500 to gamble on roulette at the casino. His stake at each game is £1, which he bets on 'odds' (so he will win if any of the 18 odd numbers comes up). If 'odds' comes up he wins £1 and gets his stake back. If 'odds' does not come up he loses his £1 stake. He decides to play until he loses all his money or has doubled his money to £1000. Find the probability that Kermit doubles his money.

Repeat the question if he stakes £10 at each game and receives an additional £10 if he wins the game.

2. (From one of exam papers) Gonzo plays a series of independent games. At the start of each game he pays £1 then rolls a fair 6-sided die. If he obtains a 6 he receives £C; otherwise he receives nothing. The games continue until he throws a 1, when the series of games stop. Find the expected amount he wins and hence state the value of C for which the game is fair (the game is said to be fair if the expected amount he wins is zero).

Part 2

3.A. Two dice are thrown repeatedly until either 2 sixes are obtained or the product of the numbers is six. The product from the final throw is recorded. Let E be the event that the recorded number is 6. By conditioning on the outcome of the first throw of the two dice, use the Total Probability Formula to obtain $P(E)$. 40

3.B. (From one of exam papers) A lift moves at random up and down between floors in a building with 10 floors, including the ground floor, numbered $0, 1, ..., 9$. Except when the lift reaches either the top or ground floor, each time it reaches a floor it is equally likely to go either up or down one floor. 30

Gonzo enters the lift on the third floor (floor 3). Find the probability that he reaches the top floor (floor 9) before he reaches the ground floor (floor zero).

3.C. Kermit has £50 to gamble on roulette at a casino with no zero, so that $P('odds') = \frac{1}{2}$. 30
His stake at each game is £1, which he bets on 'odds'. If 'odds' comes up he wins £1 and gets his stake back. If 'odds' does not come up he loses his £1 stake. He decides to play until he loses all his money or has doubled his money to £100. Find $E[T]$, where T counts the number of games he plays.

Obtain $E[T]$ if his stake at each game is £5. Nothing else is altered.