MTH5118 Probability II. Problem Sheet 1.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY PLEASE

This coursework consists of two parts. **You are required to submit solutions to the second part only. However, you are strongly encouraged to solve all problems on this problem sheet.** *Please staple your coursework and post it in the Blue Box in the basement of the Maths building by* **17:00** *on Tuesday 6rd October 2009.*

Part 1

1. For each of the following functions $G_X(t)$ state (with reasons) if it can be the probability generating function of a discrete random variable X which takes non-negative integer values and (if appropriate) determine the probability mass function of X:

(a) $G_X(t) = \frac{1}{2} + \frac{1}{4}t + \frac{3}{4}t^3$; (b) $G_X(t) = \frac{1}{4}(1+t^2)^2$; (c) $G_X(t) = \frac{2}{(1+t)}$.

Hint: recall the simplest properties and the graph of $G_X(t)$.

2. X is a random variable with probability generating function $G_X(t)$. In each of the following cases state the probability distribution of X, i.e. name the distribution and specify its parameters: (a) $G_X(t) = \left(\frac{3}{4} + \frac{1}{4}t\right)^3$; (b) $G_X(t) = \frac{1}{64}(3+t)^3$; (c) $G_X(t) = e^{5t-5}$.

Hint: We shall show in our first lecture on Monday that if X is Binomial(n,p) then the associated p. g. f. is $G_X(t) = (q+pt)^n$. Meanwhile, you can use this fact to answer the above question.

3. Let X be a random variable with probability generating function $G_X(t) = \frac{t}{3-2t}$. Name the distribution of X, including any parameter values. Using $G_X(t)$, find $P(X=1)$, $P(X=2)$, $E(X)$ and $\mathrm{Var}(X)$.

4. Let X and Y be two independent random variables, $X \sim$ Bernoulli(1/2) and $Y \sim$ Bernoulli(2/3). Write down the probability generating functions $G_X(t)$ and $G_Y(t)$. Hence obtain the probability generating function of $Z = X+Y$ and use this to derive the probability mass function of $Z = X+Y$ (i.e. find $P(Z=n)$ for the values of n for which this probability is positive).

Hint: we shall prove that if X and Y are independent random variables, then $G_Z(t) = G_X(t)G_Y(t)$. Meanwhile, you can use this property to solve Problem 4 as well as the problems in Part 2.

Part 2

5.A. Let X and Y be independent random variables each with Poisson distribution, with parameters λ and μ respectively. Show that $Z = X + Y$ has Poisson distribution and state its parameter. If $X_1, X_2, ..., X_n$ are independent identically distributed random variables, with common distribution which is Poisson with parameter λ, find the probability generating function of $W = \sum_{j=1}^{n} X_j$ and hence state the distribution of W. 30

5.B. Let X be a random variable with probability generating function $G_X(t) = \frac{t(1+t)}{2(3-2t)}$. Using $G_X(t)$, find $E(X)$ and $\mathrm{Var}(X)$. 30

5.C. Factor $G_X(t)$ given in 5.B into the product of two probability generating functions $G_X(t) = G_Y(t)G_Z(t)$, hence proving that X may be expressed in the form $X = Y + Z$ where Y and Z are independent random variables. Name the distributions of Y and Z and their parameters. Use results for the mean and variance of the sum of two independent random variables Y and Z to obtain $E(X)$ and $\mathrm{Var}(X)$ (which should be identical to the results obtained above). 40