MTH4108 Probability 1 2009/10 Exam Numerical Solutions

Q1 (b) (i)
$$\{(x, y, z) : 1 \le x, y, z \le 5, x \ne y \ne z \ne x\}.$$

(ii)
$$\{(2, y, 4) : y \in \{1, 3, 5\}\}.$$

(iii) $T^c \cap F$ is the event that 4 is chosen and 2 is not chosen.

$$\mathbb{P}(T \cup F^c) = 18/60 = 3/10.$$

Q2 (c)
$$\mathbb{P}(A \cup B) = 1/2 + 1/2 - 1/4 = 3/4$$
.

$$Q3$$
 (b) (i) F and H are independent.

(ii)
$$F \cap H$$
 and J are not independent.

(iii) F, H and J are not mutually independent.

Q4 (b)
$$\mathbb{E}(X) = 7/4$$

$$Var(X) = 27/16.$$

Q5 (c) (i)
$$\mathbb{P}(X \ge 2) = 5/32$$

$$\mathbb{P}(X \ge 2 \,|\, X \ge 1) = 10/37.$$

(ii)
$$3/7$$

$$\mathbb{P}(X = x \text{ and } Y = y) = \begin{cases} 0 & \text{if } x = y \\ 1/12 & \text{if } x \neq y \end{cases}$$

for all
$$x, y \in \{0, 1, 2, 3\}$$
.

(ii)
$$\mathbb{E}(X) = 3/2 = \mathbb{E}(Y)$$
, $\mathbb{E}(XY) = 11/6$, $Cov(X, Y) = -5/12$.