

(10 December 2008, 10.00 - 12.00, LRC2)

NOTE: You *do not* hand in your solutions for marking, but you are welcome to discuss them with me at my office hours if you wish. Also, the solutions will be given at the course website on Thursday, 11 December.

6. ARIMA(p,d,q)

6.1 Length of the thermal growing season: 1772-2006, England

(source: <http://www.defra.gov.uk/environment/statistics/globalatmos/gakf19.htm>)

The growing season is the period of time each year during which plants can grow. The thermal growing season length is defined as beginning when the temperature on five consecutive days exceeds 5°C and ending when the temperature on five consecutive days is below that threshold.

The data are given at course website in two formats: mtw and txt.

Follow the points of Chapter 7.1 to fit the best model to the data and to predict observations for next five years. Note that it may be better to consider only a part of the data, such as recent 60 or so years.

6.2 Paleoclimatic Glacial Varves.

(source: Shumway and Stoffer (2000). *Time Series Analysis and Its Applications*. Example 1.24)

Varve is defined as a layer or series of layers of sediment deposited annually in a still body of water, e.g. by a glacier. Varves can be counted back to date a specific layer.

Glacial yearly varve thickness data were collected at a location in Massachusetts for 634 years, beginning 11834 years ago. The data are given at the course website in two formats: mtw and txt.

Fit an ARIMA(p,d,q) model to the data. Predict five more values of the varve thickness.

6.2 Theory Questions

6.2.1 Classify each of the following models

(a) $X_t = 0.7X_{t-12} + Z_t + 0.5Z_{t-12}$

(b) $X_t = Z_t - 1.3Z_{t-4} + 0.4Z_{t-8}$

(c) $X_t = 0.8X_{t-1} - 0.15X_{t-2} + Z_t - 0.3Z_{t-1}$

(d) $(1 - 1.3B + 0.5B^2)(1 - B^{12})X_t = (1 + 0.5B)Z_t$

(e) $(1 - 0.8B + 0.25B^2)\nabla X_t = Z_t$

6.2.2 Show that ACF of a seasonal AR(1)₄ is

$$\rho(\tau) = \begin{cases} 1 & \text{for } \tau = 0 \\ \Phi^k & \text{for } \tau = 4k, k=1,2,\dots \\ 0 & \text{for otherwise} \end{cases}$$

Use the method of matching coefficients as in Chapter 4.6.1 for AR(1).