Queen Mary
University of London

School of Mathematical Sciences

MTH6139 TIME SERIES **Computer Lab 3**

(29 October 2008, 10.00 - 12.00, LRC2)

NOTE:

*You **do not** hand in your solutions for marking, but you are welcome to discuss them with me at my office hours if you wish. Also, the solutions will be given at the course website on Monday, 3rd November.*

3.1 India Monsoon Data

Download the data from STU directory and do the following:

1. Draw the time series plot of the data.
2. Detrend the data using the differencing method and draw the time series plot of the detrended data.
3. Draw the sample ACF of the detrended data. What kind of a model does the sample ACF suggest?
4. Use the option Stat → Time Series → ARIMA to fit an appropriate model. Note that ARIMA(p,0,q) ≡ ARMA(p,q). As the model diagnostic tools use plots of the residuals and their sample ACF as well as the tests given in the Minitab output (T-test for the parameters of the model and LBQ-test for the residuals' autocorrelations). Find out in Help what the LBQ test is.
5. Put the graphs and relevant numerical results into your report. Write down the form of the fitted model and your comments. Note that Minitab uses $-\theta$ in place of θ in our model notation.

3.2 Boston Crime data

Do the same points as for the India Monsoon data, but in point 2 use Stat → Time Series → Trend Analysis… to fit an appropriate trend.

3.3 Theory questions

3.3.1 Derive the ACVF and ACF of MA(2): $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2},$ where $Z_t \sim WN(0,\sigma^2)$.

3.3.2 Show that ACF of MA(q) of the form

$$X_t = \frac{1}{q+1}\sum_{k=0}^{q} Z_{t-k}$$

is

$$\rho(\tau) = \begin{cases} \dfrac{q+1-\tau}{q+1}, & \tau = 0,1,\ldots,q \\ 0, & \tau > q \end{cases}$$

3.3.3 Given a seasonal series of monthly observations $\{X_t\}$, assume that the seasonal effects s_t are constant, that is $s_t = s_{t-12}$ for all t, the trend m_t is linear and also that there is a stationary zero mean random noise Y_t.

- Show that for an additive model $X_t = m_t + s_t + Y_t$ the operator ∇_{12} acting on X_t reduces the series to a stationary process.
- Does the operator ∇_{12} reduces X_t to a stationary process if the model is multiplicative, that is, if $X_t = m_t s_t + Y_t$? If not, find a differencing operator which does.

3.3.4 Give the definition of an ARMA(1,1) model. Explain what is causality and what is invertibility of a time series model. For what range of the model parameters φ and θ an ARMA(1,1) process is causal and invertible?