Chapter 1

Introduction

The following **abbreviations** are used in these notes:

TS Time Series

ma moving average filter

rv random variable

iid independently identically distributed

cdf cumulative distribution function

pdf probability density function

μ expected value of a random variable

σ^2 variance of a random variable

$N(\mu, \sigma^2)$ normal standard distribution of a random variable with expected value μ and with variance σ^2

AR(p) Autoregressive model of order p

MA(q) Moving Average model of order q

ARMA(p,q) Autoregressive Moving Average model of order (p, q)

ARIMA(p,d,q) Autoregressive Integrated Moving Average model of order (p, q) with the integration parameter d.

Also, CAPITAL Latin letters are used to denote random variables, small Latin letters are used to denote realization of the random variables.

1.1 Introductory Definitions and Examples

A **Time Series** (TS) is a collection of observations made sequentially, usually in *time*, but the observations may be collected in other domain as well, in another kind of *distance*.
We denote a TS of a variable X as follows

$$X = (X_1, X_2, \dots, X_t, \dots)$$

or

$$X = \{X_t\}_{t=1,2,\dots}$$

TS theory finds applications in a variety of fields. For example in

Economics: X = Unemployment, Consumption;

Meteorology: X = Changes in global temperature, Summer monsoon rainfall in India;

Sciences: X = Chemical process temperature;

Sport: X = Olympic gold medal performance in track and field events.

TS has following features

1. It can be continuous or discrete,
   - continuous TS is when observations are made continuously,
   - discrete TS is when observations are made at specific times, usually equispaced (every second, every day etc); TS is then discrete even if the measured variable is continuous, like *river level* measured every day.
2. Successive observations are usually correlated, so future values may be predicted from past values.
3. TS can be deterministic or stochastic,
   - it is deterministic if the future observations may be predicted exactly,
   - it is stochastic when such exact prediction is impossible due to randomness of the observations. Then future can only be partly determined by past.

In this course we will consider **discrete stochastic time series**.

How do we analyze a TS? We can do it by:

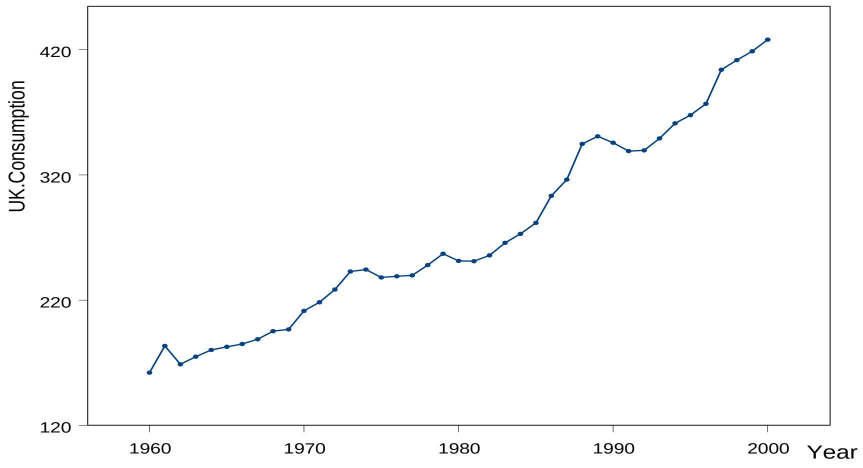

Figure 1.1: Yearly values of consumption in the UK, 1960-2000, Billions of pounds sterling, 1990 prices. Source: http://www.fgn.unisg.ch/eumacro/macrodata/macroeconomic-time-series.html

Description: Plot of the observations in time gives a general view of the phenomenon variable X represents. It shows what kind of variation occurs in time, is there any trend or seasonality, are there any unusual observations (outliers), or turning points.

Explanation: Construction of a mathematical model which explains the observed variability in the data.

Prediction: Based on the model we can predict, with some confidence, future observations.

Control: If the predicted future values are 'off target' then it may be possible to change the factors which influence the observed variable and so to control the outcome.

1.2 Simple Descriptive Techniques

1.2.1 The Time Plot

A good picture is better than 1000 words.

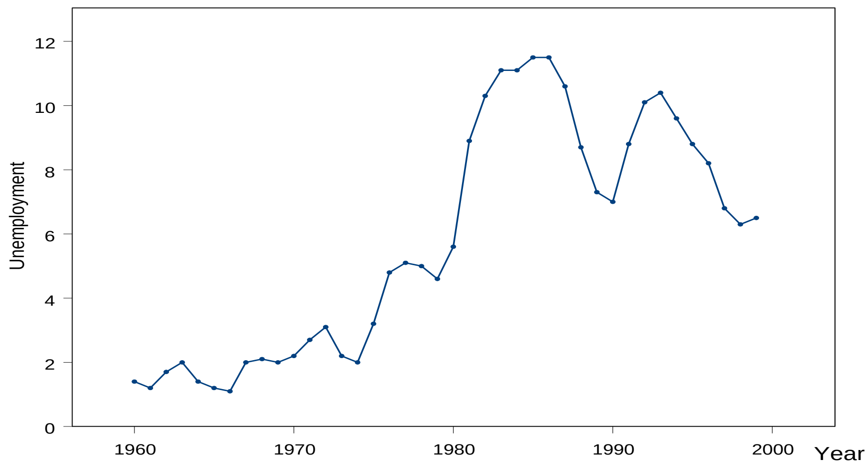

Figure 1.2: Yearly values of unemployment percentage in the UK, 1960-1999. Source: http://www.fgn.unisg.ch/eumacro/macrodata/macroeconomic-time-series.html

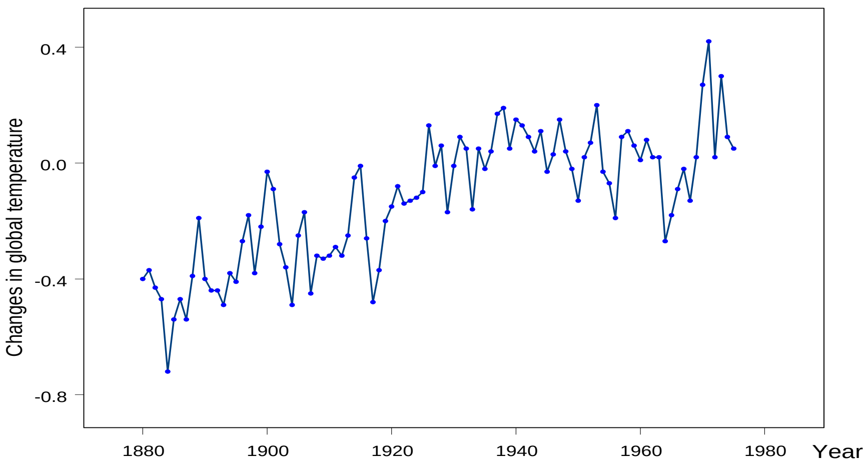

Figure 1.3: Surface air 'temperature change' for the globe, 1880-1985. Degrees Celsius. 'Temperature change' means temperature against an arbitrary zero point. Source: J.Hansen and S.Lebedeff (1987).

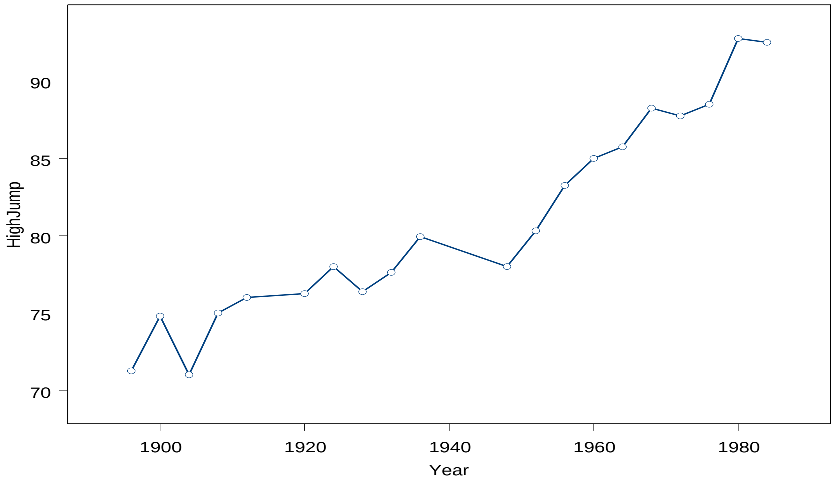

Figure 1.4: The gold medal performance in the men's high jump (measured in inches) for the modern Olympic series, 1896-1984. Source: http://lib.stat.cmu.edu/DASL

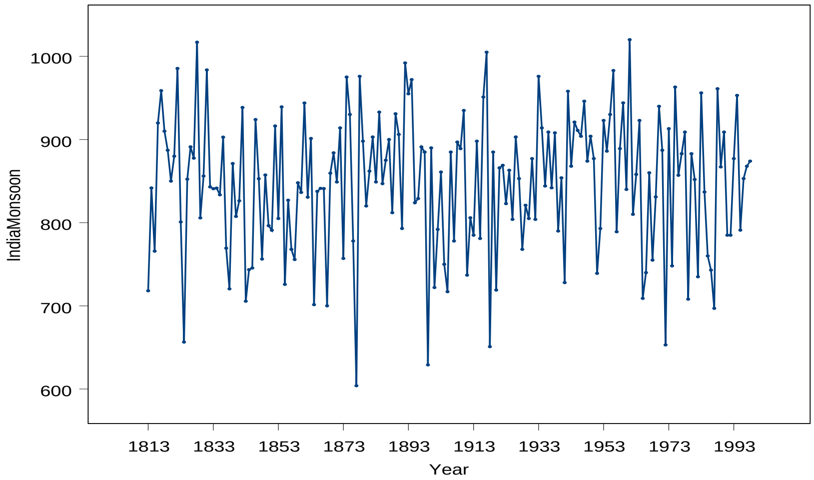

Figure 1.5: Summer monsoon rainfall data from India. Units mm. Jun-Sep 1813 to Jun-Sep 1998.

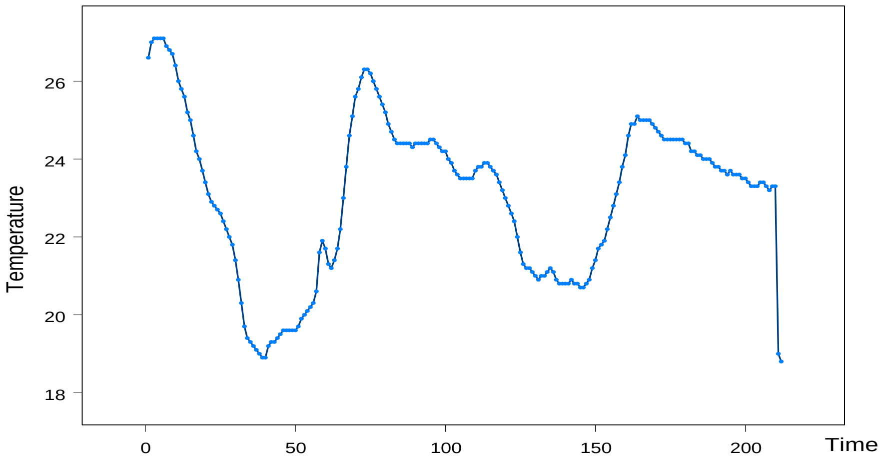

Figure 1.6: Chemical process temperature readings, every minute. Degrees Celsius. Source: Box and Jenkins (1976).

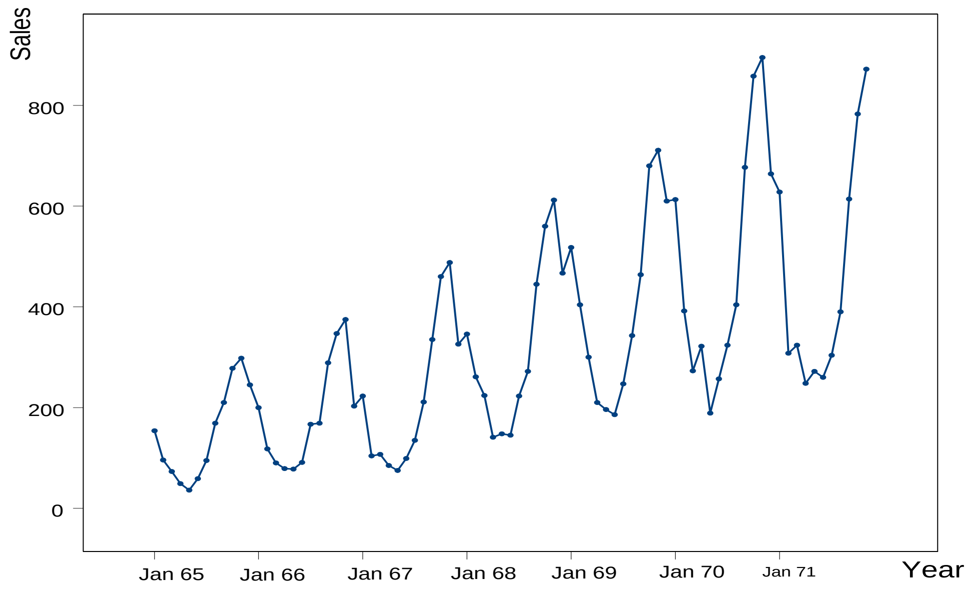

Figure 1.7: Sales of an industrial heater; monthly data starting from January 1965 till December 1971. Source: Chatfield (2004). (The last value is added for completeness).

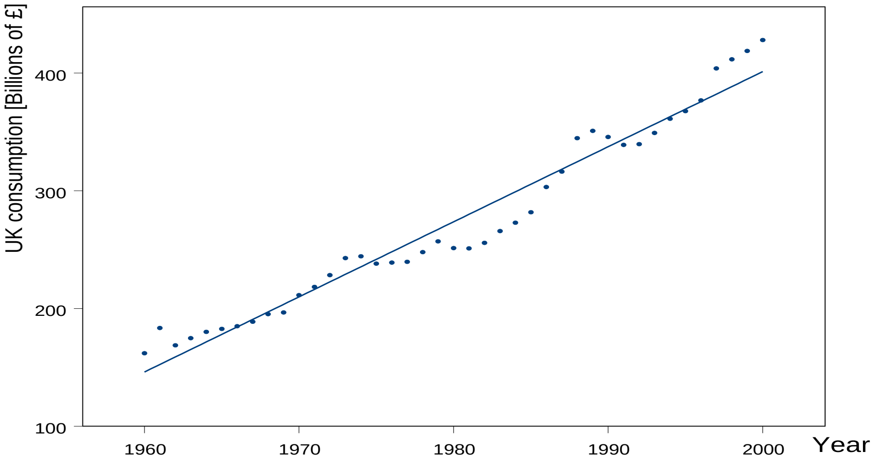

Figure 1.8: TS and a linear trend for the UK consumption data, compare Fig. 1.1

A plot representing the observations against *time* gives an initial analysis of the data. A good TS plot should

- give a clear and self-explanatory title,
- state units of measurements,
- have carefully chosen scales, including the size of intercept,
- have clearly labelled axes,
- have appropriate plotting symbol and line (if a line is included).

A TS plot may reveal various features of the data, such as

- Trend, which indicates a long term change in the mean level. For example, UK consumption data indicate a steady growth in time, a straight line would well describe the trend of growth.
- Periodicity, which shows a pattern repeating in time. Sales of an industrial heater data have both trend and periodicity (see Figure 1.7).
- Unusual features, such as a sudden single peak or a turning point. For example a sudden drop of the temperature of the chemical process at the end of measurement time (see Figure 1.6).

1.2.2 Deterministic Trend and Seasonality

A deterministic trend model with a seasonal effect can take either an additive form,

$$X_t = m_t + s_t + Y_t, \qquad t = 0, 1, \ldots, n, \tag{1.1}$$

or a multiplicative form, such as

$$X_t = m_t s_t Y_t, \qquad t = 0, 1, \ldots, n, \tag{1.2}$$

or a mixed form,

$$X_t = m_t s_t + Y_t, \qquad t = 0, 1, \ldots, n, \tag{1.3}$$

where $m_t = m(t)$ is a (usually slowly changing) function of time, so called 'trend component', $s_t = s(t)$ is a periodical function of time and Y_t is a random noise component. Model (1.2) can be easily transformed to the additive form by taking a logarithm of both sides. Model (1.3) is often referred to as a multiplicative one.

The most common trend function is a polynomial of a degree $k \geq 1$, i.e.,

$$m(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \ldots + \beta_k t^k \tag{1.4}$$

A linear trend $m(t) = \beta_0 + \beta_1 t$ is a special case of (1.4).